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Comparison of ARCH-GARCH and stochastic approaches for estimating volatility. Application to a small stock market

Comparación de los enfoques ARCH-GARCH y estocástico para estimar la volatilidad. Aplicación a un pequeño mercado de valores

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ABSTRACT

Economic time series often exhibit volatility, where the variance of the observational error fluctuates over time. One of the most widely used methodologies for modeling these dynamics is the ARCH model, introduced by Engle (1982), and its extensions, such as GARCH models. These assume that conditional variance depends on past values of the series. In contrast, stochastic volatility models (SVM), first proposed by Taylor (1980, 1986), assume that volatility depends on past variances but not directly on past returns. This study compares both approaches in modeling the volatility of a small stock market. To evaluate the performance of ARCH-GARCH and stochastic volatility models in estimating market risk and identifying volatility patterns in the Merval index, which represents the Buenos Aires Stock Exchange (BASE). A longitudinal observational study was conducted using daily Merval index data from January 13, 2003, to May 22, 2015, covering 3006 observations. This period was chosen to avoid political shifts that could introduce market distortions. Statistical tests (ADF, Phillips-Perron) were performed to check stationarity, and models were estimated using maximum likelihood and Kalman filtering. GARCH models with heavy-tailed distributions provided better short-term volatility predictions, capturing volatility clustering, while stochastic volatility models were more effective at identifying regime shifts. The Merval index, with an average market capitalization of $312 million, confirms the characteristics of a small stock market, where volatility models play a crucial role in risk assessment. The choice between ARCH-GARCH and stochastic models depends on the forecasting horizon. GARCH models are optimal for short-term risk evaluation, whereas stochastic models are better suited for detecting long-term structural changes. Combining both approaches enhances volatility modeling in low-liquidity markets.

Keywords: Volatility, ARCH-GARCH models, State space models, Kalman filter, Stochastic volatility, Merval index.

RESUMEN

Las series temporales económicas suelen presentar volatilidad, lo que implica que la varianza del error de observación fluctúa con el tiempo. Una de las metodologías más utilizadas para modelar estas dinámicas es el modelo ARCH, introducido por Engle (1982), y sus extensiones, como los modelos GARCH. Estos suponen que la varianza condicional depende de valores pasados de la serie. En contraste, los modelos de volatilidad estocástica (SVM), propuestos inicialmente por Taylor (1980, 1986), asumen que la volatilidad depende de las varianzas pasadas pero no directamente de los rendimientos previos. Este estudio compara ambos enfoques en la modelización de la volatilidad en un mercado bursátil pequeño. Evaluar el desempeño de los modelos ARCH-GARCH y de volatilidad estocástica en la estimación del riesgo de mercado e identificación de patrones de volatilidad en el Merval index, que representa la Bolsa de Comercio de Buenos Aires (BASE). Se realizó un estudio longitudinal observacional basado en datos diarios del Merval index entre el 13 de enero de 2003 y el 22 de mayo de 2015, abarcando 3006 observaciones. Se seleccionó este período para evitar cambios en la afiliación política del gobierno, eliminando posibles distorsiones exógenas del mercado. Se aplicaron pruebas estadísticas (ADF, Phillips-Perron) para verificar estacionariedad y los modelos fueron estimados mediante máxima verosimilitud y filtrado de Kalman. Los modelos GARCH con distribuciones de colas pesadas predijeron mejor la volatilidad en el corto plazo, capturando el clustering de volatilidad, mientras que los modelos estocásticos fueron más eficaces en la detección de cambios de régimen. El Merval index, con una capitalización promedio de $312 millones, confirma las características de un mercado bursátil pequeño, donde la modelización de la volatilidad es clave para la evaluación del riesgo. La elección entre modelos ARCH-GARCH y estocásticos depende del horizonte de pronóstico. Los modelos GARCH son óptimos para evaluar el riesgo en el corto plazo, mientras que los modelos estocásticos son más adecuados para detectar cambios estructurales a largo plazo. La combinación de ambos enfoques mejora la modelización de la volatilidad en mercados de baja liquidez.

Palabras claves: Volatility, ARCH-GARCH models, State space models, Kalman filter, Stochastic volatility, Merval index.

1 INTRODUCTION

The study of the phenomenon of volatility has been developed mainly from the analysis of time series related to the economy. However, it must be emphasized that any time series may be subject to the presence of volatility.

Many economic time series do not have a constant mean and in practical situations we often see that the variance of the observational error, conditional on past knowledge, is subject to substantial variability over time. This phenomenon is known as *volatility*.

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To take into account the presence of volatility in an economic series it is necessary to resort to models known as conditional heteroscedastic models. In these models, the variance of a series at a given moment of time depends on past information and other data available up to that time, so a conditional variance must be defined, which is not constant and does not coincide with the overall variance of the observed series.

An important characteristic of financial time series is that they are not generally serially correlated, but rather dependent. Thus, linear models such as those belonging to the ARMA model family may not be appropriate to describe these series.

There is a very large variety of non-linear models in the literature, useful for the analysis of economic time series with volatility. An important class of them are the ARCH-type models introduced by Engle (1982) and its extensions. These models are non-linear with respect to variance.

The ARCH or GARCH family of models assume that the conditional variance (volatility) depends on past observations. In other words, if σ_t^2 is the volatility, the ARCH-GARCH family assumes that it depends on the series y_j for $j < t$. On the other hand, the *stochastic volatility model* or SVM, first proposed by Taylor (1980, 1986), is not based on this assumption. This model assumes that the volatility σ_t^2 depends on its past values (σ_j^2 for $j < t$) but is independent of the past of the series under analysis (y_j for $j < t$).

A cursory inspection of series such as the one presented in this paper suggests that they do not have a constant mean and variance. A stochastic variable in which the variance is constant is said to be *homoscedastic* as opposed to a *heteroscedastic* variable. For those series in which there is volatility, the unconditional variance may be constant even though the conditional variance in some periods is unusually large and in others small.

As an application, the Merval index series is analyzed. The Merval is a stock market index that has been calculated in the Buenos Aires Stock Exchange (BASE), Argentina, since June 30, 1986. It measures the traded volume of the main shares listed on that exchange. The index is composed of a fixed nominal amount of shares of different listed companies, commonly known as “leading companies”. The shares that make up the Merval index change every three (3) months, when this portfolio is recalculated, based on the participation in the traded volume and the number of operations of the last six (6) months. Those shares that are within the accumulated 80% of market participation are selected. In addition, the selected companies must meet the requirement of having traded in at least 80% of the trading sessions of the period considered.

The Buenos Aires Stock Exchange was founded on July 10, 1854. It is the largest stock exchange and the main business and financial centre of the Argentine Republic. Its transactions are basically shares of important national and foreign companies, bonds, currencies and futures contracts. It is a non-profit civil association run by representatives of the various business sectors. According to a study carried out by the International Finance Corporation, which is part of the World Bank, the average value of the companies listed on the BASE is 312 million dollars, a figure that places Argentina in 30th place among the countries that have stock markets. That is why we say that we are dealing with a small stock market.

As stated, in this paper we analyze the Merval index series. This is a series with information corresponding to all working days of the stock market. Specifically, we work with the returns of the quotes of this index, which consists of the first differences of the logarithm of the Merval levels. The period analyzed goes from January 13, 2003 to May 22, 2015. There are 3,006 observations. It covers a period in which there was no change in the government's affiliation. In fact, during that period the wing of Peronism called Kirchnerism governed. This eliminates the effects that could have been introduced into the market by changes in the governing group.

To perform the analysis we used two approaches: one based on ARCH-GARCH type models, and another based on stochastic volatility models. We then made comparisons between these two approaches.

2 ECONOMIC AND FINANCIAL TIME SERIES MODELLING

The basic idea of a time series is very simple, it consists of the recording of any fluctuating quantity measured at different points in time.

Specifically, a *time series* is a set of observations $\{y_1, ..., y_n\}$ ordered in time. The basic and general model used to represent any time series is the additive model, given by

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad t = 1, ..., n, \tag{1}$$

wehere μ_t is a component that changes smoothly over time called *trend*, γ_t is a periodic component called *seasonality* and ε_t is an irregular component called *error*. As we can see, the common feature of all records belonging to the time series domain is that they are influenced, at least partially, by sources of random variation.

The main reason for modelling a time series is to enable prediction of its future values. The distinctive feature of a time series model, as opposed to, for example, an econometric model, is that no attempt is made to formulate a behavioural relationship between the time series under consideration and other explanatory variables. Movements of the series are explained solely in terms of its own past, or by its position relative to time or by its structure. Predictions are made by extrapolation.

Many economic time series do not have a constant mean and in many cases there are periods of relative calm followed by periods of significant changes. Much of the current research in time series and econometrics is focused on extending the classical and commonly used methodology of Box and Jenkins to analyze this type of behaviour. However, there is a characteristic present in time series that refer to financial assets (or directly financial time series) and other series referring to economic activities and it is what is known as *volatility*, which can be defined in various ways, but is not directly observable. To take into account the presence of volatility groups in a financial or economic series it is necessary to resort to models known as *conditional heteroscedastic models*. In these models, the variance (or volatility) of a series at a given time depends on its past and other information available up to that time, so a *conditional variance* must

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be defined, which is not constant and does not coincide with the global or non-conditional variance of the observed series.

3 THE VOLATILITY

Volatility is defined as the variance of a random variable, conditional on all past information. Since volatility cannot be measured directly, it can manifest itself in various ways in a time series.

Let y_t be the series under study whose dimension is $p = 1$. We define

$$\mu_t = E(y_t\,|Y_{t-1}) = E_{t-1}(y_t), \tag{2}$$

$$\begin{aligned}\sigma_t^2 &= var(y_t\,|Y_{t-1}) = E\left\{(y_t-\mu_t)^2\,|Y_{t-1}\right\} \\ &= E_{t-1}(y_t-\mu_t)^2 = var_{t-1}(y_t),\end{aligned} \tag{3}$$

as the conditional mean and variance of y_t given the information up to time $t-1$ contained in Y_{t-1}, respectively.

A common model to take volatility into account is of the form

$$y_t = \mu_t + \sqrt{h_t}\varepsilon_t, \tag{4}$$

where $E_{t-1}(\varepsilon_t) = 0$, $var(\varepsilon_t) = 1$ and typically the ε_t are independent and identically distributed (iid) with distribution function F. The unconditional mean and variance of y_t will be denoted as $\mu = E(y_t)$ and $\sigma^2 = var(y_t)$, respectively, and let G be the distribution function of y_t. It is clear that (2), (3) and F determine μ, σ^2 and G, but not the opposite. More details about this formulation can be seen in Abril M. (2014).

4 MODELS OF THE ARCH-GARCH FAMILY

There is a very large variety of non-linear models available in the literature to deal with volatility, but we will concentrate on the ARCH type models or *autoregressive with conditional heteroscedasticity models*, introduced by R. Engle (1982) and its extensions. These models are non-linear as far as the variance is concerned.

In the analysis of non-linear models the errors (also called innovations, because they represent the new part of the series that cannot be predicted from the past) ε_t, are generally assumed to be iid and the model has the form

$$\begin{aligned}y_t &= g(\varepsilon_{t-1},\varepsilon_{t-2},\ldots) + \varepsilon_t h(\varepsilon_{t-1},\varepsilon_{t-2},\ldots) \\ &= g_t + \varepsilon_t h_t = \mu_t + \varepsilon_t h_t,\end{aligned} \tag{5}$$

where $g(\cdot) = g_t = \mu_t$ represents the conditional mean and $h^2(\cdot) = h_t^2$ is the conditional variance. If $g(\cdot)$ is non-linear, the model is said to be *non-linear in the mean*, on the other hand if $h(\cdot)$ is non-linear, the model is said to be *non-linear in the variance*. For example, the model

$$y_t = \varepsilon_t + \alpha\varepsilon_{t-1}^2,$$

is non-linear in the mean since $g(\cdot) = \alpha\varepsilon_{t-1}^2$ and $h(\cdot) = 1$, while the model

$$y_t = \varepsilon_t\sqrt{\alpha y_{t-1}^2},$$

is non-linear in variance since $g(\cdot) = 0$ and $h(\cdot) = \sqrt{\alpha y_{t-1}^2}$ and y_{t-1} depends on ε_{t-1}.

ARCH models or autoregressive with conditional heteroscedasticity models were first introduced by Engle in 1982 to estimate the variance of inflation in Britain. The basic idea of this model is that the price of an asset y_t is not serially correlated but depends on past prices via a quadratic function.

In conventional econometric models, the variance of the disturbance is assumed to be constant. However, it can be seen that many economic series exhibit periods of unusually large volatility followed by periods of relative calm. In these circumstances, the assumption of constant variance, also called homoscedasticity, is somewhat inappropriate. There are instances when one wishes to predict the conditional variance of a series. Asset holders may be interested in predictions of the rates of return and their variance for a given period. The unconditional variance (i.e. the long-term variance) would not be important if one plans to buy the asset at time t and sell it at $t+1$.

An ARCH(q) model can be expressed as

$$y_t = \mu_t + \varepsilon_t h_t = \mu_t + \varepsilon_t\sigma_t, \quad \varepsilon_t \sim \text{iid } D(0,1), \tag{6}$$

$$\sigma_t^2 = h_t^2 = \omega + \sum_{i=1}^{q}\alpha_i z_{t-1}^2, \tag{7}$$

where $z_t = y_t - \mu_t$ and $D(\cdot)$ is a probability density function with mean equal to zero and unit variance.

An ARCH model adequately describes *volatility clustering*. The conditional variance of y_t is an increasing function of the square of the shock occurring at time $t-1$. Consequently, if y_{t-1} is large enough in absolute value, σ_t^2 and thus y_t are expected to be large enough in absolute value as well. It should be noted that even though the conditional variance in an ARCH-type model varies over time, i.e., $\sigma_t^2 = E\left(z_t^2\,|Y_{t-1}\right)$ the unconditional variance of z_t is constant and, since $\omega > 0$ and $\sum_{i=1}^{q}\alpha_i < 1$, we have

$$\sigma^2 \equiv E\left\{E\left(z_t^2\,|Y_{t-1}\right)\right\} = \frac{\omega}{1-\sum_{i=1}^{q}\alpha_i}. \tag{8}$$

If ε_t is normally distributed, then $E\left(\varepsilon_t^3\right) = 0$ and $E\left(\varepsilon_t^4\right) = 3$. Therefore, $E\left(z_t^3\right) = 0$ and the symmetry of the variable z will be equal to zero. Thus, the kurtosis coefficient for an ARCH(1) is $3(1-\alpha_1^2)/(1-3\alpha_1^2)$ if $\alpha_1 < \sqrt{1/3} \approx 0,577$. In this case, the conditional distribution of any series will have heavy tails if $\alpha_1 > 0$.

In most practical applications, excessive kurtosis in an ARCH model means that a normal distribution is not adequate enough to explain the process generating the data. Therefore, we can make use of other distributions. For example, we can assume that ε_t follows a Student t distribution with mean 0, variance equal to 1 and υ degrees of freedom, that is, ε_t is $ST(0,1,\upsilon)$. In this case, the unconditional kurtosis for the ARCH(1) is $\lambda(1-\alpha_1^2)/(1-\lambda\alpha_1^2)$ where $\lambda = 3(\upsilon-2)/(\upsilon-4)$. Due to the additional coefficient υ, the ARCH(1) model based on a t distribution will have heavier tails than the one based on a normal distribution, which will be very useful when analyzing the data in our study. It is important to note that other distributions

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for performing the analysis are available in some software packages.

The calculation of σ_t^2 in (7) depends on the past unobserved quadratic residuals, z_t^2, for $t = 0, -1, \ldots, -q+1$.To initialize the process, the unobserved quadratic residuals are set to a value equal to the sample mean of the observed ones.

The conditional mean μ_t can contain $n_1$explanatory variables, which are specified as follows

$$\mu_t = \mu + \sum_{i=1}^{n_1} \delta_i \, x_{i,t}. \tag{9}$$

On the other hand, n_2 explanatory variables can be included in the conditional variance given in (7), as follows

$$\omega_t = \omega + \sum_{i=1}^{n_2} \omega_i \, x_{i,t}. \tag{10}$$

where the $x_{i,t}$ of (9) are not necessarily the same as those appearing in (10).

σ_t^2 must obviously be positive for all t. The sufficient conditions that ensure that the conditional variance is positive in (7) are given by $\omega > 0$ and $\alpha_i \geqslant 0$ for all i. Furthermore, when explanatory variables enter the ARCH specification, these positivity restrictions no longer hold, although we still require that the conditional variance be non-negative.

A very simple device for reducing the number of parameters to be estimated, which we will not develop in this paper, is called *variance orientation* and was first developed by Engle and Mezrich (1996).

The conditional variance matrix in an ARCH model, and in most of its generalizations, can be expressed in terms of its unconditional variance and other parameters. By this means, it is possible to reparameterize the model using the unconditional variance and replace it by a consistent estimator before maximizing the likelihood.

Applying variance orientation for an ARCH model involves replacing ω by $\sigma^2 \left(1 - \sum_{i=1}^{q} \alpha_i\right)$, where σ^2 is the unconditional variance of y_t, which can be consistently estimated by its sample counterpart.

If explanatory variables appear in the ARCH equation then ω is replaced by

$$\sigma^2 \left(1 - \sum_{i=1}^{q} \alpha_i\right) - \sum_{i=1}^{n_2} \omega_i \overline{x}_i,$$

where $\overline{x}_i$ is the sample average of the variable $x_{i,t}$, assuming that there is stationarity in the n_2 explanatory variables. In other words, the explanatory variables are centered.

While Engle (1982) certainly made the major contribution to financial econometrics, ARCH-type models are rarely used in practice due to their simplicity. A good generalization of these models is found in the GARCH-type models introduced by Bollerslev (1986). These models are also a weighted average of the past squared residuals, are more parsimonious than ARCH-type models and even in their simplest form have proven to be extremely successful in predicting conditional variances.

It should be noted that GARCH-type models are not the only extension of ARCH-type models and there are at least twelve specifications related to them that will be the subject of future research.

The generalized ARCH models (or GARCH models as they are also known) are based on an infinite ARCH specification and allow reducing the number of parameters to be estimated by imposing non-linear restrictions on them. The GARCH(p, q) model is expressed as follows

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i z_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \tag{11}$$

Using the lag operator L, the GARCH(p, q) model is transformed into

$$\sigma_t^2 = \omega + \alpha\left(L\right) z_t^2 + \beta\left(L\right) \sigma_t^2,$$

where $\alpha\left(L\right) = \alpha_1 L + \alpha_2 L^2 + \ldots \alpha_q L^q$ and $\beta\left(L\right) = \beta_1 L + \beta_2 L^2 + \ldots \beta_p L^p$.

If all the roots of the polynomial $|1 - \beta\left(L\right)| = 0$ lie outside the unit circle we have

$$\sigma_t^2 = \omega \left|1 - \beta\left(L\right)\right|^{-1} + \alpha\left(L\right) \left|1 - \beta\left(L\right)\right|^{-1} z_t^2, \tag{12}$$

which can be seen as an ARCH(∞) model since the conditional variance depends linearly on all previous quadratic residuals. In this case, the conditional variance of y_t can be larger than the unconditional variance given by

$$\sigma^2 \equiv E\left(z_t^2\right) = \frac{\omega}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j},$$

if past realizations of z_t^2 are greater than σ^2 (Palm, 1996).

Applying the variance orientation procedure to a GARCH model involves replacing ω by $\sigma^2 \left(1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j\right)$, where σ^2 is the unconditional variance of z_t^2 which can be consistently estimated by means of its sample counterpart.

On the other hand, if the explanatory variables appear in a GARCH-type formulation, ω is then replaced by $\sigma^2 \left(1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j\right) - \sum_{i=1}^{n_2} \omega_i \, \overline{x}_i$, where $\overline{x}_i$ is the sample mean of the variable $x_{i,t}$, assuming stationarity of the n_2 explanatory variables.

Bollerslev (1986) showed that for a GARCH$(1, 1)$ with normal innovations, the kurtosis of y is $3\left[1 - \left(\alpha_1 + \beta_1\right)^2\right] \big/ \left[1 - \left(\alpha_1 + \beta_1\right)^2 - 2\alpha_1^2\right] > 3$. The autocorrelations of z_t^2 were derived by Bollerslev (1986). For a stationary GARCH$(1, 1)$

$$\begin{aligned}
\rho_1 &= \alpha_1 + \left[\alpha_1^2 \beta_1 \big/ \left(1 - 2\alpha_1 \beta_1 + \beta_1^2\right)\right], \\
\rho_k &= \left(\alpha_1 + \beta_1\right)^{k-1} \rho_1, \text{ for all } k = 2, 3, \ldots.
\end{aligned}$$

In other words, the autocorrelations decline exponentially with a decline factor equal to $\alpha_1 + \beta_1$.

As in the case of ARCH models, it is necessary to impose some restrictions on σ_t^2 to ensure that it is positive for all t. Bollerslev (1986) showed that ensuring that $\omega > 0$, $\alpha_i \geq 0$

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(for $i = 1, \ldots, q$) and $\beta_j \geq 0$ (for $j = 1, \ldots, p$) are sufficient to ensure that the conditional variance is positive. In practical situations, the parameters in a GARCH-type model are estimated without using positivity restrictions. Nelson and Cao (1992) stated that imposing the condition that all coefficients are non-negative is too restrictive and that some of them may in practical situations be negative while the conditional variance remains positive (reviewing a good number of real-life situations). They consequently relaxed this fact and established sufficient conditions for the GARCH$(1, q)$ and GARCH$(2, q)$ cases based on the infinite representation given in (12). Indeed, we can see that the conditional variance is strictly positive provided $\omega \left|1 - \beta(L)\right|^{-1}$ is positive and all coefficients of the polynomial $\alpha(L)\left|1 - \beta(L)\right|^{-1}$ in (12) are nonnegative. The positivity restrictions proposed by Bollerslev (1986) can be fixed during the estimation. If not, they, as well as those imposed during the ARCH(∞), representation, will be tested a posteriori if there is no explanatory variable in the conditional variance equation.

4.1 Distributions and estimation

There are basically three methods for estimating ARCH-GARCH type models:

1. The most commonly used one, and the one we will work with in this case, is the standard estimation by maximum likelihood. This uses Newton's quasi-maximum likelihood method developed by Broyden, Fletcher, Goldfrab and Shanno (or BFGS according to its acronym).
2. An optimization technique that implements quadratic sequence programming to maximize a nonlinear function, subject to nonlinear constraints similar to Algorithm 18.7 in Nocedal and Wright (1999). This is particularly useful when imposing positivity or stationarity constraints such as $\alpha_1 > 0$ in an ARCH model.
3. And finally a simulation algorithm to optimize non-smooth functions with multiple possible local maxima.

In our case, the estimation in ARCH-GARCH type models is carried out using the quasi-maximum likelihood method, so it is necessary to make an additional assumption about the innovation process ε_t, that is, to choose the density function $D(0,1)$ that has a mean equal to zero and a unitary variance.

Weiss (1986) and Bollerslev and Woolridge (1992) showed that under the assumption of normality, the quasi-maximum likelihood estimator (or QML according to its acronym) is consistent if the conditional mean and the conditional variance are correctly specified. This estimator is, however, inefficient with an increasing degree of inefficiency as it deviates from that normality (Engle and González-Rivera 1991).

As stated by Palm (1996), Pagan (1996) and Bollerslev, Chou and Kroner (1992), the use of heavy-tailed distributions is widespread in the literature. Bolleslev (1987), Hsieh (1989), Baillie and Bolleslev (1989) and Palm and Vlaar (1997) among others, showed that these distributions perform better when capturing higher order kurtosis.

For our problem, we will consider three distributions when approaching the estimation process; the normal distribution, the Student t distribution and the skewed-Student distribution.

The logic of the maximum likelihood method is to interpret the density as a function of the set of parameters, conditional on the set of sample observations. This function is called the *likelihood function*. It is evident from (8) that the recursive evaluation of this function is conditional on the observed values. For this reason, we will consider the approximate or conditional maximum likelihood and not the exact maximum likelihood.

The logarithm of the likelihood function for the standard normal distribution is given by

$$l_{norm} = -\frac{1}{2}\sum_{t=1}^{n}\left[\log(2\pi) + \log\left(\sigma_t^2\right) + \varepsilon_t^2\right], \qquad (13)$$

where n is the number of observations.

For a Student t distribution this function is

$$\begin{aligned} l_{Stud} &= n\left\{\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log\left[\pi(\nu-2)\right]\right\} \\ &\quad -\frac{1}{2}\sum_{t=1}^{n}\left[\log\left(\sigma^2\right) + (1+\nu)\log\left(1 + \frac{\varepsilon_t^2}{\nu-2}\right)\right], \qquad (14) \end{aligned}$$

where ν are the degrees of freedom, with $2 < \nu \leq \infty$ and $\Gamma(\cdot)$ is the gamma function.

For a skewed Student distribution (with mean zero and variance one) this function is

$$\begin{aligned} l_{SkSt} &= n\left\{\log\Gamma\left(\frac{\eta+1}{2}\right) - \log\Gamma\left(\frac{\eta}{2}\right) - \frac{1}{2}\log\left[\pi(\eta-2)\right]\right. \\ &\quad \left. + \log\left(\frac{2}{\xi + \frac{1}{\xi}}\right) + \log(s)\right\} \\ &\quad -\frac{1}{2}\sum_{t=1}^{n}\left\{\log(\sigma_t^2) + (1+\eta)\left[1 + \frac{(s\varepsilon_t + m)^2}{\eta - 2}\xi^{-2I_t}\right]\right\}, \qquad (15) \end{aligned}$$

where

$$I_t = \begin{cases} 1 \text{ if } \varepsilon_t \geq -\frac{m}{s} \\ -1 \text{ if } \varepsilon_t < -\frac{m}{s} \end{cases},$$

ξ is the asymmetry parameter, η are the degrees of freedom of the distribution,

$$m = \frac{\Gamma\left(\frac{\eta-1}{2}\right)\sqrt{\eta-2}}{\sqrt{\pi}\,\Gamma\left(\frac{\eta}{2}\right)}\left(\xi - \frac{1}{\xi}\right),$$

and

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}.$$

It is important to note that this last function is the one that will be used when analyzing our data set.

There are other distributions that can be used to carry out the estimation process, such as the generalized error distribution or GED, but we will not develop them in our work since they are not objects of our research.

In terms of the estimation process, we can say that many authors have proposed using a Student t distribution or a skewed

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Student distribution in combination with a GARCH type model to adequately model the heavy tails in economic or financial time series whose data are of high frequency, which will be seen later.

5 MODELS FOR STOCHASTIC VOLATILITY

We will say that the series y_t follows a stochastic volatility model (SVM) if

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t, && (16)\\ \sigma_t &= e^{\frac{h_t}{2}}, && (17)\end{aligned}$$

where ε_t is a stationary series with mean equal to zero and variance one, and h_t is another stationary series with probability density given by a function $f(h)$.

As we can see in (17), h_t is not equal to the volatility σ_t^2 as is usually the notation in the ARCH-GARCH family models.

The simplest formulation of the model assumes that the logarithm of volatility is given by

$$h_t = \alpha_0 + \alpha_1 h_{t-1} + \eta_t, \qquad (18)$$

where η_t is a stationary, Gaussian series, with mean zero, variance σ_η^2 and independent of ε_t. It follows from this that we must have $|\alpha_1| < 1$.

5.1 Other SVM formulations

Other SVM formulations can be found in the literature, among which we highlight the following:

1. Canonical form of Kim, Shephard and Chib (1998). In this case the SVM is written as

$$\begin{aligned} y_t &= \beta e^{\frac{h_t}{2}} \varepsilon_t, && (19)\\ h_t &= \mu + \alpha_1(h_{t-1} - \mu) + \sigma_\eta \eta_t, && (20)\end{aligned}$$

with

$$h_t \sim N\left(\mu, \frac{\sigma_\eta^2}{1-\alpha_1^2}\right), \qquad (21)$$

where ε_t and η_t are both $N(0,1)$ and independent of each other. If $\beta = 1$, then $\mu = 0$.

2. The Jaquier, Polson and Rossi (1994) form of the SVM is equal to

$$\begin{aligned} y_t &= \sqrt{h_t}\varepsilon_t, && (22)\\ \log(h_t) &= \alpha_0 + \alpha_1 \log(h_{t-1}) + \sigma_\eta \eta_t, && (23)\end{aligned}$$

where ε_t and η_t are both $N(0,1)$ and independent of each other.

5.2 Properties of SVM

Let us return to the model defined in the equations (16), (17) y (18). Suppose that $\{\varepsilon_t\}$ constitutes a succession of independent random variables such that $\varepsilon_t \sim N(0,1)$, then $\log(\varepsilon_t^2)$ has a distribution called “log chi square”, such that

$$\begin{aligned} E\{\log(\varepsilon_t^2)\} &\approx -1,27, && (24)\\ var\{\log(\varepsilon_t^2)\} &= \pi^2/2. && (25)\end{aligned}$$

From (16), (17) and (18) we get

$$\begin{aligned} \log(y_t^2) &= \log(\sigma_t^2) + \log(\varepsilon_t^2), && (26)\\ h_t &= \log(\sigma_t^2) = \alpha_0 + \alpha_1 h_{t-1} + \eta_t. && (27)\end{aligned}$$

Calling $\xi_t = \log(\varepsilon_t^2) - E\{\log(\varepsilon_t^2)\} \approx \log(\varepsilon_t^2) + 1,27$, we have that $E(\xi_t) = 0$, $var(\xi_t) = \pi^2/2$ and

$$\begin{aligned} \log(y_t^2) &= -1,27 + h_t + \xi_t, && \xi_t \sim \text{iid } (0, \pi^2/2), (28)\\ h_t &= \alpha_0 + \alpha_1 h_{t-1} + \eta_t, && \eta_t \sim \text{iid } N(0, \sigma_\eta^2) (29)\end{aligned}$$

where iid means that the variables are independent and identically distributed. It is also assumed that ξ_t and η_t are independent of each other at all times.

From the equations (16), (17) and (18) let us calculate some moments of the SVM. Taking expectation of (16) we have

$$E(y_t) = E(\sigma_t \varepsilon_t) = E(\sigma_t)E(\varepsilon_t) = 0, \qquad (30)$$

given that σ_t and ε_t are independent.

The variance of y_t is

$$var(y_t) = E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) = E(\sigma_t^2)E(\varepsilon_t^2) = E(\sigma_t^2). \quad (31)$$

Since we assume that $\eta_t \sim N(0, \sigma_\eta^2)$ and that h_t is stationary with

$$E(h_t) = \frac{\alpha_0}{1-\alpha_1} = \mu_h, \qquad (32)$$

and with

$$var(h_t) = \frac{\sigma_\eta^2}{1-\alpha_1^2} = \sigma_h^2, \qquad (33)$$

we have that

$$h_t \sim N\left(\frac{\alpha_0}{1-\alpha_1}, \frac{\sigma_\eta^2}{1-\alpha_1^2}\right). \qquad (34)$$

Since h_t is normal or Gaussian, $\sigma_t^2 = e^{h_t}$ is log-normal, then we have

$$var(y_t) = E(y_t^2) = E(\sigma_t^2) = e^{\mu_h + \sigma_h^2/2}. \qquad (35)$$

It is not difficult to show that

$$E(y_t^4) = 3e^{2\mu_h + 2\sigma_h^2}, \qquad (36)$$

from which we obtain that the kurtosis of y_t is

$$K(y_t) = \frac{3e^{2\mu_h + 2\sigma_h^2}}{e^{2\mu_h + \sigma_h^2}} = 3e^{\sigma_h^2} > 3, \qquad (37)$$

as expected, that is, there are heavy tails for the SVM.

The autocovariance function of the series y_t is given by

$$\gamma_y(s) = E(y_t y_{t+s}) = E(\sigma_t \sigma_{t+s} \varepsilon_t \varepsilon_{t+s}) = 0, \qquad (38)$$

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since ε_t and η_t are independent. Then y_t is serially uncorrelated but not independent since there is correlation in $\log(y_t^2)$. Let us denote as $z_t = \log(y_t^2)$, then the autocovariance function of the series z_t is given by

$$\gamma_z(s) = E[(z_t - E(z_t))(z_{t+s} - E(z_{t+s}))]. \tag{39}$$

As the first term in parentheses of (39) is equal to $h_t - E(h_t) + \xi_t$ and h_t is independent of ξ_t, we get

$$\begin{aligned}\gamma_z(s) &= E[(h_t - E(h_t) + \xi_t)(h_{t+s} - E(h_{t+s}) + \xi_{t+s})] \\ &= E[(h_t - E(h_t))(h_{t+s} - E(h_{t+s}))] \\ &\quad + E(\xi_t \xi_{t+s}),\end{aligned} \tag{40}$$

and calling $\gamma_h(s)$ and $\gamma_\xi(s)$ respectively to the autocovariances of the right hand of (40), we have

$$\gamma_z(s) = \gamma_h(s) + \gamma_\xi(s), \tag{41}$$

for all s.

As we are assuming that (18) is satisfied, that is, we have a AR(1) model, we get

$$\gamma_h(s) = \alpha_1^s \frac{\sigma_\eta^2}{1-\alpha_1^2}, \quad s > 0. \tag{42}$$

Besides $\gamma_\xi(s) = 0$ for $s > 0$. Then, $\gamma_z(s) = \gamma_h(s)$ for all $s \neq 0$. With this we can write the autocorrelation function of $z_t = \log(y_t^2)$ as

$$\rho_z(s) = \frac{\gamma_z(s)}{\gamma_z(0)} = \frac{\alpha_1^s \sigma_\eta^2/(1-\alpha_1^2)}{\gamma_h(0) + \gamma_\xi(0)}, \quad s > 0, \tag{43}$$

from which we get

$$\rho_z(s) = \frac{\alpha_1^s}{1 + \frac{\pi^2}{2\sigma_\eta^2}}, \quad s > 0,$$

which tends to zero exponentially from the lag $s = 2$, and this indicates that $z_t = \log(y_t^2)$ can be modeled using an AR(1) model.

En la práctica obtenemos valores de α_1 próximos de uno, lo que implica la aparición de altas correlaciones para la volatilidad y consecuentes grupos de volatilidades en la serie.

A general SVM can be obtained if an AR(p) model is admitted for h_t, that is

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t, & (44)\\ \sigma_t &= e^{\frac{h_t}{2}}, & (45)\\ (1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p) h_t &= \alpha_0 + \eta_t, & (46)\end{aligned}$$

where the lag operator is defined as $B^j h_t = h_{t-j}$, the assumptions about the innovations ε_t and η_t are the same as those made previously, but now we assume that the roots of the polynomial $(1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p)$ are outside the unit circle.

The SVM have been extended to include the fact that volatility has long memory, in the sense that the autocorrelation function of $z_t = \log(y_t^2)$ slowly decays, although as we saw in this case, the y_t have no serial correlation.

5.3 Estimation of the SVM

SVM models are difficult to estimate. We can use the approach proposed by Durbin and Koopman (1997a, 1997b, 2000, 2001, 2012) which consists of using a quasi-maximum likelihood procedure by means of the Kalman filter and smoother. In this case, the model defined in the equations (16), (17) y (18) can be re-expressed in the form

$$\begin{aligned} y_t &= \sigma \varepsilon_t e^{\frac{h_t}{2}}, & (47)\\ \sigma_t &= \sigma e^{\frac{h_t}{2}}, & (48)\\ h_t &= \alpha_1 h_{t-1} + \eta_t, & (49)\end{aligned}$$

where $\sigma = \exp(\alpha_0/2)$ is a scale factor, α_1 is a parameter, and η_t is a disturbance term which in the simplest model is uncorrelated with ε_t. Literature reviews of this model were carried out by Shephard (1996, 2005) and Ghysels, Harvey and Renault (1996). This SVM has two main attractions. The first is that it is a discrete-time natural (Euler) analogue of the continuous-time model used in option pricing work, such as that of Hull and White (1987). The second is that its statistical properties are easy to determine. The disadvantage with respect to conditional variance models of the GARCH type is that likelihood-based estimation can only be performed by computationally intensive techniques such as those described in Kim, Shephard and Chib (1998) and Sandmann and Koopman (1998). However, a quasi-maximum likelihood method is relatively easy to implement and is usually reasonably efficient. The method is based on writing (47), (48) y (49) in the following equivalent form

$$\begin{aligned} \log\left(y_t^2\right) &= \kappa + h_t + \xi_t, & (50)\\ h_t &= \alpha_1 h_{t-1} + \eta_t, & (51)\end{aligned}$$

where $\xi_t = \log\left(\varepsilon_t^2\right) - E\{\log\left(\varepsilon_t^2\right)\}$ and $\kappa = \log\left(\sigma^2\right) + E\{\log\left(\varepsilon_t^2\right)\}$.

The equations (50) and (51) are expressed in the form of state space, as can be seen in Abril and Abril (2018). The formula (50) is called the *observation equation* or *measurement equation* and the formula (51) is called the *state equation* or *transition equation*. The estimation process is therefore carried out using the Kalman filter and smoother in the same way as those developed in the previous reference.

It is important to make some observations here:

1. When α_1 in (51) is close to 1, the fit of an SVM is similar to that of a GARCH$(1,1)$ model with the sum of its coefficients close to 1.

2. When $\alpha_1 = 1$ in (51), h_t is a random walk and the fit of an SVM is similar to that of an IGARCH$(1,1)$ model.

3. When some observations are equal to zero, which can occur in practice, the logarithmic transformation specified in (50) cannot be performed. One way to avoid this problem is to subtract the overall mean of the series y_t of each of the observations and taking this result as the series to work on;

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that is, taking as a working series

$$y_t - \overline{y}, \quad t = 1, \ldots, n, \tag{52}$$

where $\overline{y} = n^{-1} \sum_{t=1}^{n} y_t$. Another solution, suggested by Wayne Fuller and analyzed by Breidt and Carriquiry (1996), is to make the following transformation based on a Taylor expansion

$$\log\left(y_t^2\right) = \log\left(y_t^2 + cS_y^2\right) - \frac{cS_y^2}{y_t^2 + cS_y^2}, \quad t = 1, \ldots, n, \tag{53}$$

where S_y^2 is the sample variance of the series y_t and c is a small number. Versions prior to 8.3 of the STAMP program developed by Koopman, Harvey, Doornik, and Shephard incorporated the transformation defined in (53) with $c = 0,02$ as a pre-specified operation that could be used if needed. Starting with version 8.3 of that program (see Koopman, Harvey, Doornik, & Shephard, 2010) that transformation was no longer a pre-specified element, and that or other transformations could be performed, such as the one defined in (52), by using the calculator or the availability of Algebra within that program according to the user's requirements and needs.

As shown in Harvey, Ruiz and Shephard (1994), the state space form given by the equations (50) and (51) provides the basis for quasi-maximum likelihood estimation via the Kalman filter and smoother and also allows for constructing smoothed estimates of the component h_t of the variance and make predictions. One of the attractions of the quasi-maximum likelihood approach is that it can be applied without an assumption about a particular distribution for ε_t. Another attraction of using a quasi-maximum likelihood procedure using Kalman filter and smoother to estimate SVM is that it can be carried out directly using standard computing packages such as STAMP by Koopman, Harvey, Doornik, and Shephard (2010). This is a major advantage compared to more labor-intensive simulation-based methods.

Shephard and Pitt (1997) proposed the use of *importance sampling* to estimate the likelihood function in the non-Gaussian case.

Since the SVM is a hierarchical model, Jaquier, Polson, and Rossi (1994) proposed a Bayesian analysis of the model. See also Shephard and Pitt (1997) and Kim, Shephard, and Chib (1998). An overview of the SVM estimation problem is provided by Motta (2001).

5.4 Series with errors following a SVM with structural components

The basic SVM given in (47), (48) and (49) captures only the salient features of changing conditional heteroscedasticity in a time series. In some cases the model is more accurate when the series y_t is modeled by incorporating structural components, explanatory variables and other characteristics that explain its behavior, all of this done through a state space scheme with errors that follow a SVM with structural components, for example with seasonality. Based on the above, for a univariate series y_t, this can be formulated as

$$y_t = \mathbf{Z}_t \beta_t + \nu_t, \tag{54}$$

$$\beta_t = \mathbf{T}_t \beta_{t-1} + \mathbf{R}_t \omega_t, \quad \omega_t \sim N(\mathbf{0}, \mathbf{Q}_t), \quad t = 1, \ldots, n, \tag{55}$$

with

$$\nu_t = \sigma e^{\frac{h_t}{2}} \varepsilon_t, \tag{56}$$

$$\sigma_t = \sigma e^{\frac{h_t}{2}}, \tag{57}$$

$$h_t = \alpha_1 h_{t-1} + \eta_t, \tag{58}$$

where β_t is the state vector of order $m \times 1$, ω_t are serially independent disturbances, independent of each other and independent of ν_t at all times. The system matrices $\mathbf{Z}_t$, $\mathbf{T}_t$, $\mathbf{R}_t$ y $\mathbf{Q}_t$ have dimensions $1 \times m$, $m \times m$, $m \times m$ and $m \times m$ respectively, and if there are unknown elements in them, they are incorporated into the vector ψ of hyperparameters which is estimated by maximum likelihood. (54) is called measurement equation or observation equation and (55) transition equation or equation of state. The equations (54) y (55) define a state space model with all the characteristics and properties of the same presented in Abril and Abril (2018). In effect, there can be trends, seasonality, cycles, explanatory variables and other important characteristics that explain the behavior of the process $\{y_t\}$. The equations (56), (57) and (58) define an SVM with structural components (seasonality in this case) for the errors of the state space model given above, where ε_t is a stationary series with mean equal to zero and variance one, and η_t is a stationary, Gaussian series, with mean zero, variance σ_η^2 and independent of ε_t at all times.

The estimates of the state space models are performed using standard computing packages such as STAMP by Koopman, Harvey, Doornik, and Shephard (2010), which is the one used in this work.

In this case, the volatility is equal to

$$\sigma_t^2 = \sigma^2 e^{h_t}. \tag{59}$$

The practical treatment in these cases is as follows: given a series $\{y_t\}$, the linear components that can explain the behavior of its mean are identified, including the explanatory variables that may correspond, in such a way as to explicitly define the model of the equations (54) and (55).This first performs filtering and then smoothing of Kalman, obtaining the smoothed estimator $\widehat{\beta}_t$ of the the state vector β_t. This estimator allows to calculate the smoothed residuals as

$$\widehat{\nu}_t = y_t - \mathbf{Z}_t \widehat{\beta}_t, \quad t = 1, \ldots, n. \tag{60}$$

These smoothed residuals estimate the disturbances ν_t. The values of $\widehat{\nu}_t$ serve as a basis for testing the null hypothesis of lack of serial correlation of ν_t. If this hypothesis is accepted, it could be said that the model given in the equations (54) and (55) was adequately identified, defined and estimated. On the other hand, if $\log(\widehat{\nu}_t^2)$ shows serial correlation, it can be said that the errors ν_t follow an SVM of the form given in (56), (57) and (58). Therefore, it is taken to $\widehat{\nu}_t$ as the observed series and the following

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state space model is estimated

$$\log\left(\widehat{\nu}_t^2\right) = \kappa + h_t + \xi_t, \quad (61)$$
$$h_t = \alpha_1 h_{t-1} + \eta_t, \quad (62)$$

where $\xi_t = \log\left(\varepsilon_t^2\right) - E\{\log\left(\varepsilon_t^2\right)\}$, $\kappa = \log\left(\sigma^2\right) + E\{\log\left(\varepsilon_t^2\right)\}$, ε_t is a stationary series with mean equal to zero and variance one, and η_t is a stationary, Gaussian series, with mean zero, variance σ_η^2 and independent of ε_t at all times. The process of estimating (61), (62), is done using the Kalman filter and smoother.

As shown in Harvey, Ruiz and Shephard (1994), the state space form given by the equations (61) and (62) provides the basis for quasi-maximum likelihood estimation via the Kalman filter and smoother and also allows for constructing smoothed estimates of the component h_t, of the variance and make predictions. One of the attractions of the quasi-maximum likelihood approach is that it can be applied without an assumption about a particular distribution for ε_t. Another attraction of using a quasi-maximum likelihood procedure using Kalman filter and smoother to estimate SVM is that it can be carried out directly using standard computing packages such as STAMP by Koopman, Harvey, Doornik, and Shephard (2010). This is a major advantage compared to more labor-intensive simulation-based methods.

6 ANALYSIS OF THE SERIES UNDER STUDY

The data we handle belong to a very special field within statistical science, which is that of time series. The common characteristic of all records belonging to the domain of time series is that they are influenced, even if only partially, by non-observable components that contain random variations, that is, the occurrence of unplanned events.

As an application, the Merval index series is analyzed. This is a series with information corresponding to all working days of the stock market. Specifically, we work with the returns of the quotes of this index, which consists of the first differences of the logarithm of the Merval levels. The period analyzed goes from January 13, 2003 to May 22, 2015. There are 3006 observations. It covers a period in which there was no change in the government's affiliation. In fact, during that period the wing of Peronism called Kirchnerism governed. This eliminates the effects that could have been introduced in the market by changes in the governing group.

It is important to note that although this is a very long period to analyze, it is possible to carry out a very interesting study in which the main characteristics of the series can be appreciated.

First, we proceed to graph it. Within the study of a series, graphical methods are an excellent way to begin an investigation and then be able to dive into a detailed study of the subject under consideration Among the functions that tables and graphs perform are the following:

1. They make the data under study more visible, systematize and synthesize them.
2. They reveal their variations and their historical or spatial evolution.
3. They can show the relationships between the various elements of a system or process and provide clues to future correlations between two or more variables.

Furthermore, the application of these methods suggests new research hypotheses and allows the subsequent implementation of statistical models ranging from the simplest to those that are much more refined, thus achieving a better analysis of the data and its fluctuations over time.

In Figure 1 the daily series of the Merval from January 13, 2003 to May 22, 2015 is shown. The upper left box shows the levels, the upper right box shows the first differences of the logarithm of the levels called returns, the lower left box shows the histogram with the distribution of the returns compared to a normal distribution and the lower right box shows the QQ diagram of the returns. As can be seen, the returns are not normal, they have a distribution with some degree of negative asymmetry and with kurtosis. Carrying out a careful inspection of the graph of the series of returns, we can see that there are periods where the volatility is less pronounced than in others, such as the one corresponding to the year 2009.

6.1 Analysis using models from the ARCH-GARCH family

We begin the study of the Merval series by focusing first on the models of the ARCH-GARCH family and using the *Estimating and Forecasting ARCH Models Using G@RCH 7* package developed by Laurent (2013).

In Figure 2 we observe that in general the correlations and partial correlations are close to zero, except for those of order five and nineteen. This can be interpreted as the presence of two periodic components, one that coincides with the working week, which is five days long, and another with the working month, which is almost twenty days long, which leads us to think of an autoregressive model of order 20 for the conditional mean.

In Figure 3 we observe the square of the Merval returns, its distribution compared to a normal one with zero mean and variance $(0.00966)^2$, the autocorrelation function of that series and also the partial autocorrelation function of the same series under study. We see that a high-degree ARCH model or a more parsimonious GARCH model may be suitable to be applied.

Different alternatives were tested regarding the modeling of the Merval returns series for the period between January 13, 2003, and May 22, 2015. After analyzing them and seeing the values of different goodness-of-fit statistics, such as the Akaike Criterion, the Schwarz Criterion, the Schibata Criterion, or the Hannan-Quinn Criterion, we were left with a model based on the equation (5) of our work, where y_t is the series under study and its explicit specification is given by

$$y_t = \mu_t + \varepsilon_t\, h_t, \quad (63)$$

where ε_t is independent with a skewed Student distribution whose degrees of freedom are 5.82627 and the asymmetry is -0.103452. The conditional mean μ_t is equal to a general mean given by μ plus an autoregressive process of order 19 but with all coefficients equal to zero except those of order 1, 5 and 19,

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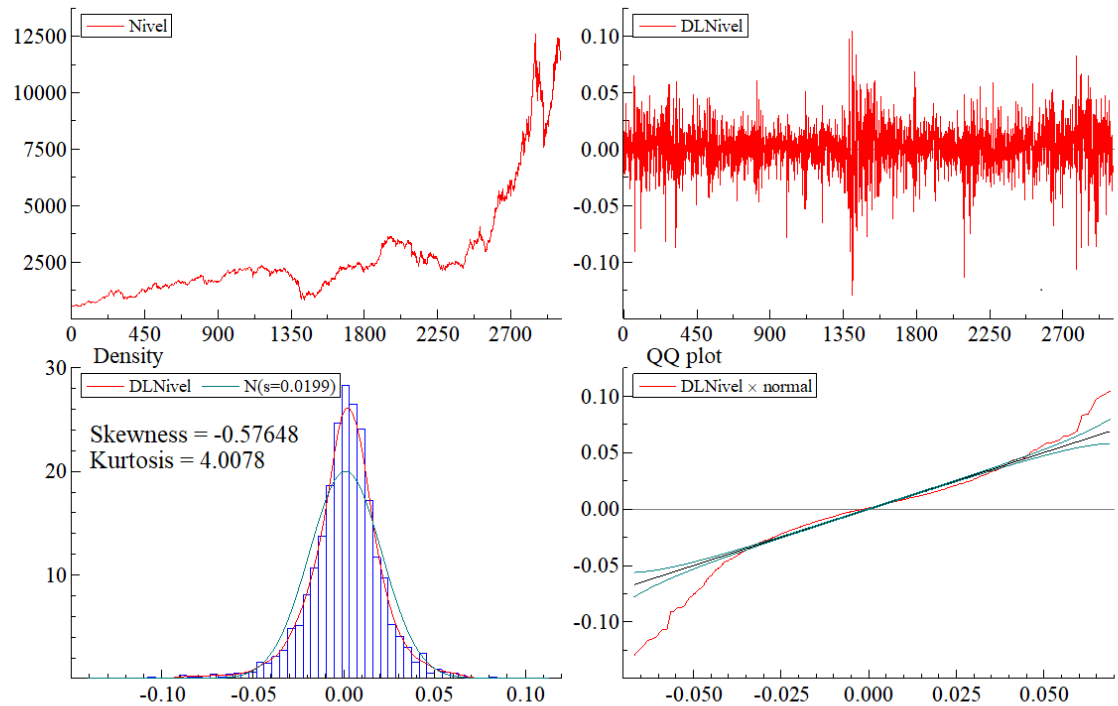
Figure 1. *Exploratory Analysis of the Merval Index (2003-2015)*



Note. The figure presents an exploratory analysis of the Merval index series from January 13, 2003, to May 22, 2015. The top left panel shows the index levels, while the top right panel displays the logarithmic first differences (returns). The bottom left panel illustrates the histogram of returns compared to a normal distribution, and the bottom right panel presents a QQ plot for assessing normality.

which is explicitly

$$\mu_t = \mu + \varphi_1 y_{t-1} + \varphi_5 y_{t-5} + \varphi_{19} y_{t-19} + \nu_t, \tag{64}$$

where $\nu_t = \varepsilon_t h_t$ of (63). Furthermore, the conditional variance in (63) is given by

$$h_t^2 = \sigma_t^2 = \omega + \alpha(y_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \tag{65}$$

that is, it is a GARCH$(1,1)$ model with a constant given by ω. The estimated values of the parameters and their corresponding standard errors for the model we have formulated in (63), (64) and (65) are expressed in the following Table 1.

In Table 1 we see that the values of the t statistic to test the null hypothesis that the coefficients φ_1 y φ_5 are equal to zero, we are led to accept this hypothesis. Consequently, models were tested in which one of the coefficients was first removed and the other was left, then they were exchanged between the one that was removed and the one that was left and finally both coefficients were removed. In all cases, the goodness-of-fit statistics or information criteria such as Akaike, Shibata, Schwarz and Hannan-Quinn gave worse results than those obtained by including these coefficients. Therefore, it was decided to leave them in the model to be estimated.

In Figure 4 we have, at the top, the conditional variance (volatility) of the Merval returns series for the period between January 13, 2003, and May 22, 2015, and the distribution of the standardized residuals after the adjustment compared with an asymmetric Student distribution with mean zero, variance one, whose degrees of freedom are 5.82627 and the asymmetry is -0.103452. From here we see that the adjustment is adequate due to the similarity between the distribution of the standardized residuals and the skewed Student distribution.

At the top of the Figure 5 the last ten observations of the series under study can be seen in blue, and the corresponding predictions (in red) of the conditional mean. The vertical bars correspond to the 95% confidence intervals that serve to compare the predicted value with that which is actually observed. In the lower part, the conditional variance corresponding to the last ten observations of our series under study is predicted. As can be seen, the predicted conditional variance, or volatility, increases smoothly over time, which is reasonable for a small market where there are usually no major changes in the values of financial assets listed on the stock exchange.

6.2 Analysis using SVM

We continue with the study of the Merval. Now using SVM. To perform the analysis and the estimates we use the STAMP computing package by Koopman, Harvey, Doornik, and Shephard (2010). It should be remembered that this program performs the estimates by quasi-maximum likelihood via the Kalman filter and smoother. After analyzing the series (see Figures **??** and **??**), and after studying several alternative models the following

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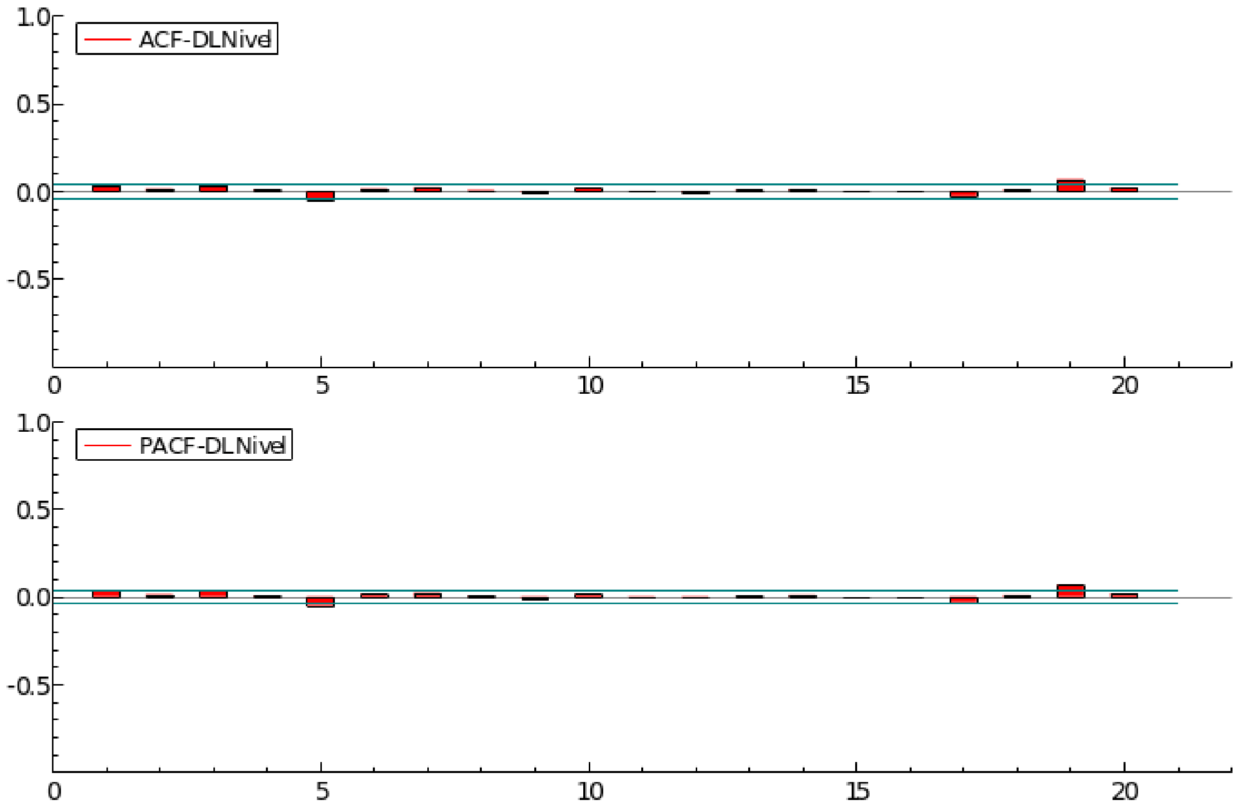
Figure 2. *Autocorrelation and Partial Autocorrelation of the Merval Returns (2003-2015)*



Note. The figure illustrates the autocorrelation (top panel) and partial autocorrelation (bottom panel) functions of the Merval returns series for the period between January 13, 2003, and May 22, 2015. These functions help identify the time dependence structure of the series.

Table 1: Estimated Values, Standard Deviations, and t-Statistics for the GARCH(1,1) Model of Merval Returns (2003-2015)

Estimator	Estimated value	Standard deviation	t value	Probability
$\widehat{\mu}$	0.001164	0.000319	3.648	0.0003
$\widehat{\varphi}_1$	0.027376	0.018400	1.488	0.1369
$\widehat{\varphi}_5$	-0.033687	0.017660	-1.908	0.0565
$\widehat{\varphi}_{19}$	0.042393	0.017573	2.412	0.0159
$\widehat{\omega}$	0.105309	0.037825	2.784	0.0054
$\widehat{\alpha}_{ARCH}$	0.092285	0.019477	4.738	0.0000
$\widehat{\beta}_{GARCH}$	0.883313	0.025945	34.05	0.0000
Asymmetry	-0.103452	0.024290	-4.259	0.0000
Degrees of freedom	5.82627	0.61778	9.431	0.0000

Note. This table presents the estimated parameters for the GARCH(1,1) model fitted to the Merval returns series from January 13, 2003, to May 22, 2015. The parameters include the mean ($\widehat{\mu}$), autoregressive coefficients ($\widehat{\varphi}$), conditional variance parameters ($\widehat{\omega}$, $\widehat{\alpha}_{ARCH}$, $\widehat{\beta}_{GARCH}$), and the asymmetry term. The degrees of freedom estimate corresponds to the assumed residual distribution.

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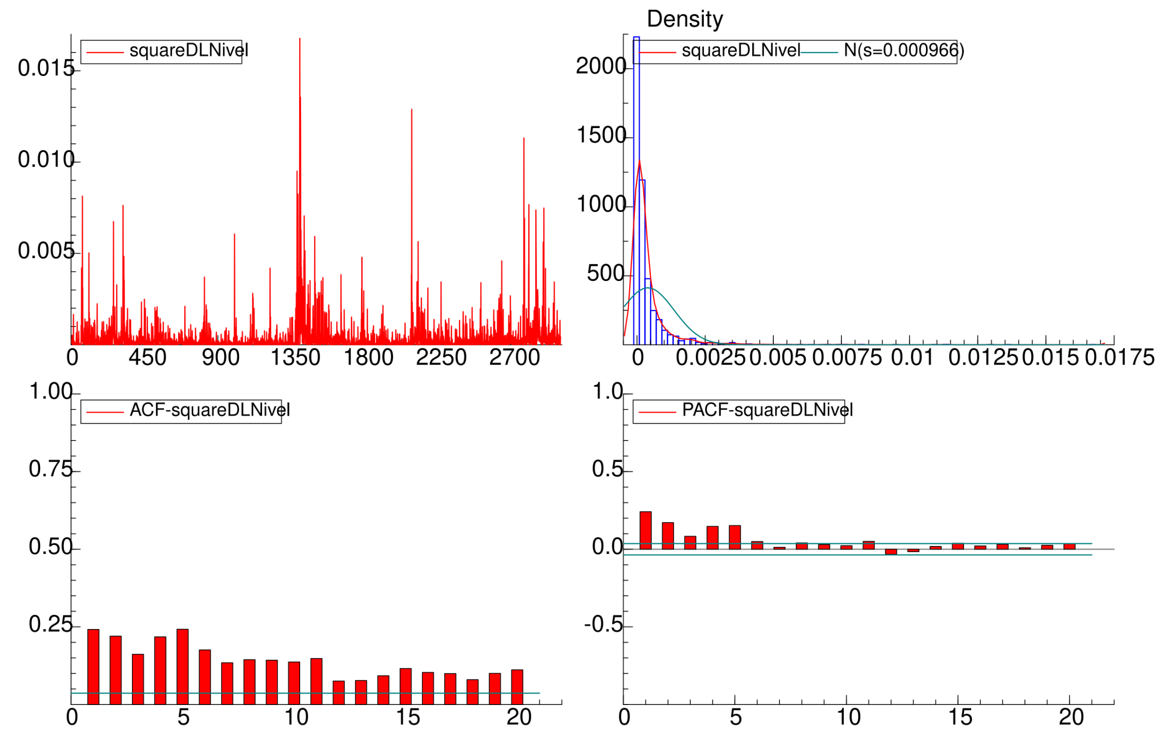
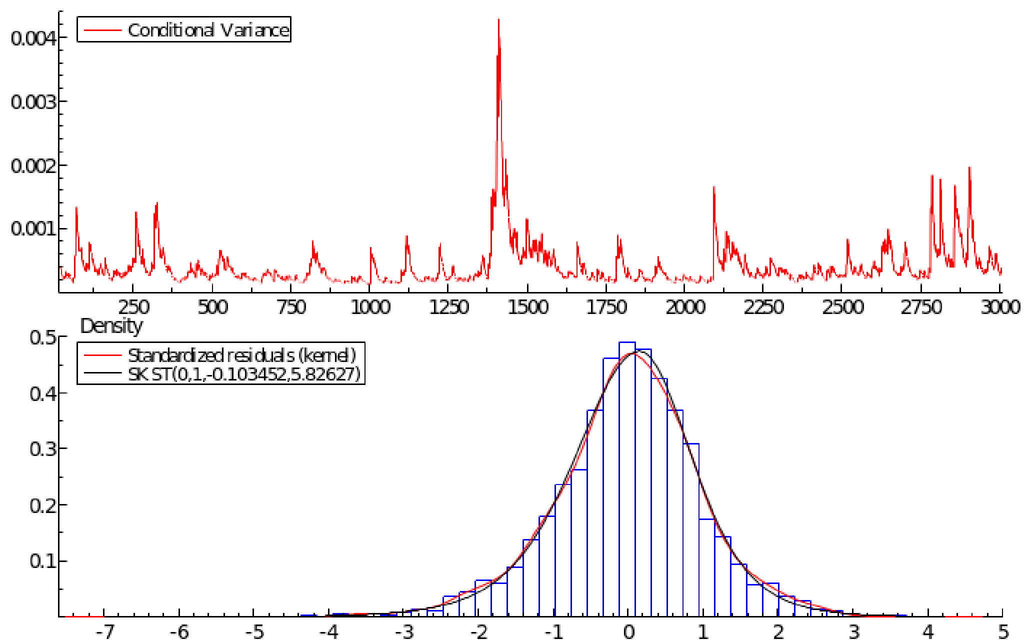
Figure 3. *Analysis of the Squared Merval Returns (2003-2015)*



Note. The figure presents an analysis of the squared Merval returns from January 13, 2003, to May 22, 2015. The top left panel shows the squared returns, while the top right panel compares their distribution to a normal one with zero mean and variance $(0.000966)^2$. The bottom left panel displays the autocorrelation function of the squared returns, and the bottom right panel presents the corresponding partial autocorrelation function.

Figure 4. *Conditional Variance and Residual Distribution of Merval Returns (2003-2015)*



Note. The top panel displays the conditional variance (volatility) of the Merval returns series from January 13, 2003, to May 22, 2015. The bottom panel shows the distribution of the standardized residuals after the adjustment, compared with a skewed Student-t distribution. The estimated degrees of freedom for this distribution are 5.82627, and the asymmetry parameter is -0.103452.

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was decided upon

$$y_t = \mu + \theta_1 y_{t-1} + \theta_5 y_{t-5} + \theta_{19} y_{t-19} + \nu_t, \tag{66}$$

with

$$\nu_t = \sigma e^{\frac{h_t}{2}} \varepsilon_t, \tag{67}$$
$$\sigma_t = \sigma e^{\frac{h_t}{2}}, \tag{68}$$
$$h_t = \alpha_1 h_{t-1} + \eta_t, \tag{69}$$

where equations (67), (68) and (69) define a SVM, ε_t is a stationary series with mean equal to zero and variance one, and η_t is a stationary, Gaussian series, with mean zero, variance σ_η^2 and independent of ε_t at all times.

This part of the study begins by estimating the model given in the equation (66). The first thing to be observed is that the Doornik-Hansen normality statistic, whose distribution under the null hypothesis of normality of the errors is a χ_2^2, gives a value of 652.86, which is very high and leads to rejecting the null hypothesis. This is not surprising since there is no Gaussianity (see Figure **??**) and volatility is present. The $H(994)$ test for heteroscedasticity, which is distributed as an $F(994, 994)$ under the null hypothesis of presence of heteroscedasticity in the series, gives a value of 1.4193, which leads to rejecting the hypothesis of existence of heteroscedasticity. Finally, the Box-Ljung q statistic, which in this case is distributed as χ_{53}^2 gives a value equal to 74.908, which leads to accepting the hypothesis of lack of serial correlation in the residuals. On the other hand, $\widehat{\sigma}_y^2 = 0.0004$, and $\widehat{\sigma}_\nu^2 = 0.00039523$.

In Table 2 the estimated values of the parameters of the equation (66) are shown, the respective standard deviations, the values of the t statistic to test the null hypothesis that the respective parameter is equal to zero and the probabilities in the tails corresponding to that hypothesis test. If these probabilities are less than 0.05, the respective hypothesis is rejected at that level of significance. As we see in these figures, except for the coefficient θ_1, all other coefficients are significantly different from zero. In the case of the coefficient θ_1, models were tested in which it was eliminated, but the goodness-of-fit tests (Akaike and others) always gave worse results than those obtained in this case in which it was included. Therefore, it was decided to continue working with this proposal and this gives us an idea that the adopted model is the appropriate one.

To study volatility, because errors ν_t of the model (66) are not observable, they are estimated by the residues of the same after the corresponding estimates and are denoted as $\widehat{\nu}_t$. The latter is the series with which we work to estimate everything related to volatility.

After being estimated the model given in (66), its standardized residuals are shown at the the upper left part of Figure 6, in the upper right part is its estimated autocorrelation function, then, in the lower left part is the estimated spectral density and finally in the lower right part is the estimated density function which is represented by a red line, compared with the normal density function which is represented by a green line. In this Figure we see that the residuals do not differ significantly from a series without serial correlation and approximately normal. Thus, for example, the estimated autocorrelations are practically within the confidence band, which implies that the respective parameters of a possible model do not differ from zero, and the oscillations of the spectral density are insignificant compared to its scale. The respective statistics also support these assertions.

To apply the state space scheme and to be able to make the respective estimates of volatility, the series of residuals must be squared and then logarithms must be taken. With this we arrive at the model given in (67), (68) and (69). It is then prepared to apply the state space scheme, that is, the following model is estimated

$$\log\left(\widehat{\nu}_t^2\right) = \kappa_t + h_t + \xi_t, \tag{70}$$
$$h_t = \alpha_1 h_{t-1} + \eta_t, \tag{71}$$

where κ_t is the stochastic level. In Figure 7 the logarithm of the squared residuals of the model given in (66) after estimation is shown at the top left, at the top right is the estimated autocorrelation function, then at the bottom left is the estimated partial autocorrelation function and finally at the bottom right is the estimated density function which is represented by a red line, compared to the normal density function which is represented by a green line. It is clearly seen there that the model given in (61) and (62) is the right one.

By estimating the model given in (61) and (62) we found that $\widehat{\sigma}^2_{\log(\widehat{\nu}_t^2)} = 5.4272$, $\widehat{\sigma}_\xi^2 = 5.12422$, $\widehat{\sigma}_\eta^2 = 0.296033$, the level κ is stochastic with variance equal to 0.00162343, and $\widehat{\kappa} = -0.89350$ at the end of the period.

The normality statistic gives a value of 624.44 which is high. This is inevitable because the transformed model (61) and (62) is not Gaussian. This should not worry us. On the other hand, the estimates of α_1 is $\widehat{\alpha}_1 = 0.91838$, which is high as expected.

In Figure 8 the logarithm of the squared residuals of the model given in (66) is shown at the top after having been estimated (black line) and the estimated level (red line), in the central part is the estimated AR(1) component whose equation is given in (62) and finally at the bottom are the estimates of the irregular component of (61).

From (68) it follows that volatility is equal to

$$\sigma_t^2 = \sigma^2 e^{h_t}, \tag{72}$$

which has two multiplicative components which are: a scale constant σ^2 and basic volatility e^{h_t}. Of these two components, the basic volatility is obviously the most important one since the other is a multiplicative constant. The basic volatility is estimated as $e^{\widehat{h}_t}$ where $\widehat{h}_t$ is

$$\widehat{h}_t = \widehat{\alpha}_1 \widehat{h}_{t-1}, \tag{73}$$

with $\widehat{\alpha}_1 = 0.91838$, which is the estimated value of α_1. To estimate σ^2, the estimated series of the irregular component of (66) corrected for basic heteroscedasticity is calculated, i.e.

$$\widetilde{\nu}_t = \widehat{\nu}_t \exp\left\{-\frac{\widehat{h}_t}{2}\right\}, \tag{74}$$

and then the variance $\widetilde{\sigma}^2$ of $\widetilde{\nu}_t$ is computed, which is an estimate

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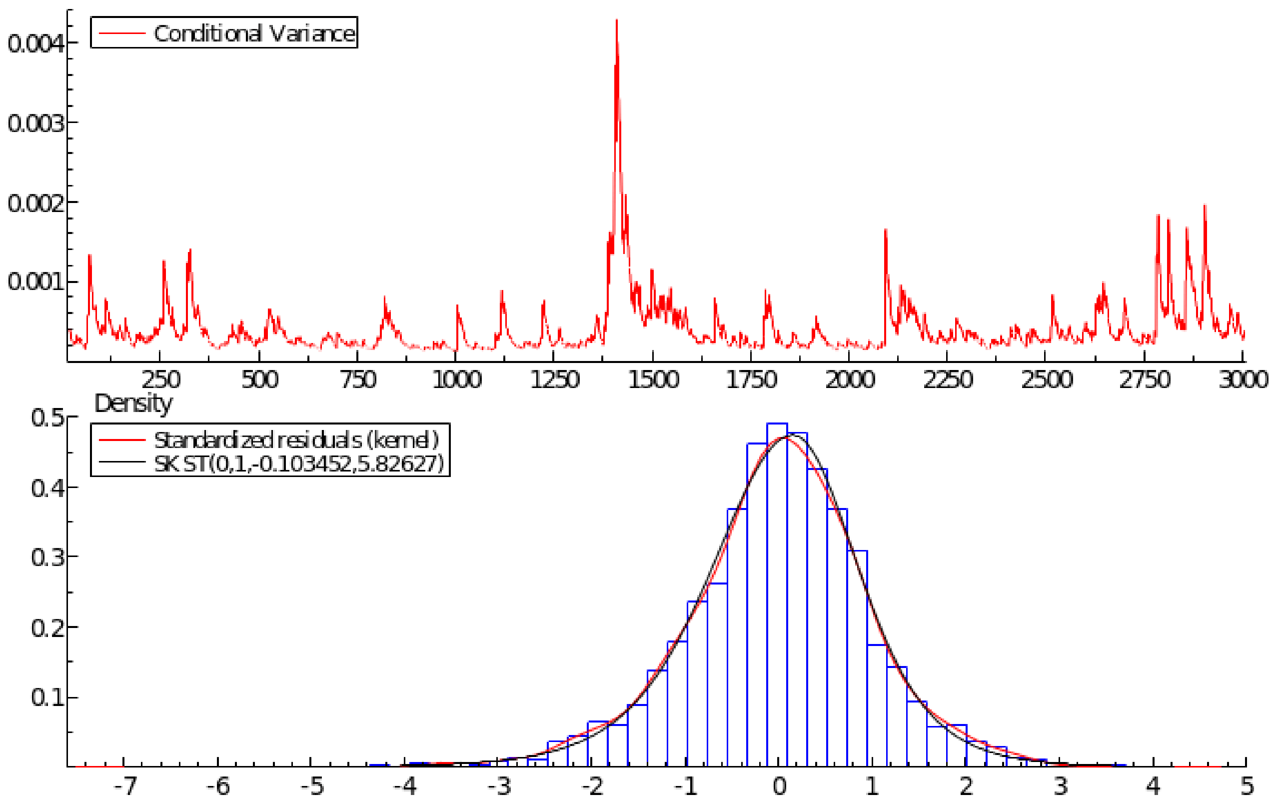
Table 2: Parameter Estimates for the Model of Merval Returns (2003-2015)

Estimator	Estimated value	Standard deviation	t value	Probability
$\widehat{\theta}_1$	0.03518	0.01824	1.92877	0.05385
$\widehat{\theta}_5$	−0.04962	0.01825	−2.71835	0.00660
$\widehat{\theta}_{19}$	0.06633	0.01824	3.63729	0.00028

Note. This table presents the estimated parameters for the model fitted to the Merval returns series from January 13, 2003, to May 22, 2015. The parameters include autoregressive coefficients ($\widehat{\theta}_1$, $\widehat{\theta}_5$, $\widehat{\theta}_{19}$), their standard deviations, t-values, and the corresponding probabilities, which indicate their statistical significance.

Figure 5. *Conditional Variance and Residual Distribution of Merval Returns (2003-2015)*



Note. The top panel displays the conditional variance (volatility) of the Merval returns series from January 13, 2003, to May 22, 2015. The bottom panel shows the distribution of the standardized residuals after the adjustment, compared with a skewed Student-t distribution. The estimated degrees of freedom for this distribution are 5.82627, and the asymmetry parameter is -0.103452.

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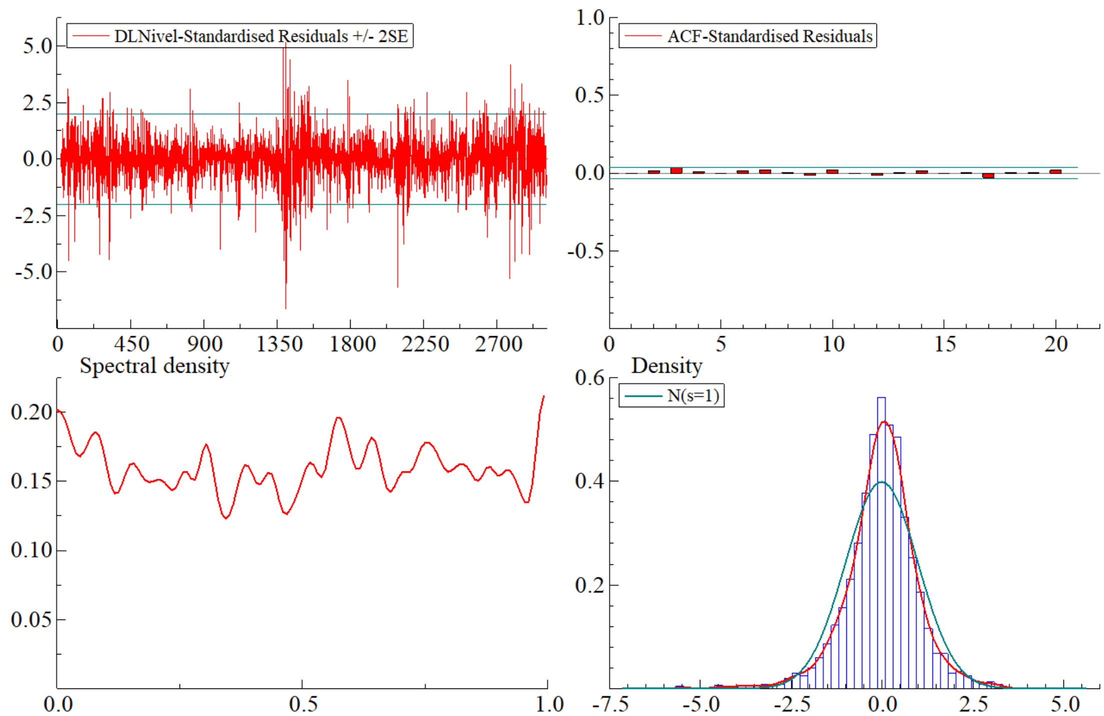
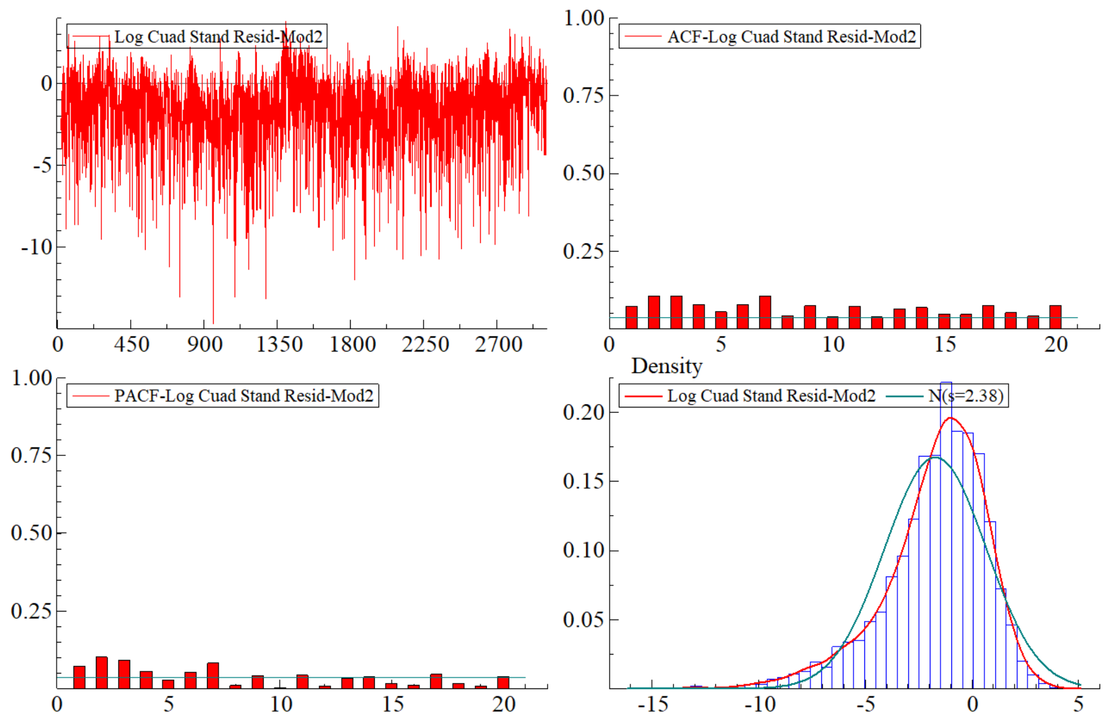
Figure 6. *Residual Analysis of the Merval Returns Model (2003-2015)*



Note. The figure presents the residual analysis of the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. The top panel displays the estimated autocorrelation function of the residuals, while the bottom panels show the estimated spectral density and the residual distribution. These diagnostics help assess the adequacy of the fitted model.

Figure 7. *Analysis of the Log-Squared Residuals of the Merval Returns Model (2003-2015)*



Note. The figure presents the analysis of the log-squared residuals of the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. The top panel displays the estimated autocorrelation function, while the middle panel presents the estimated partial autocorrelation function. The bottom panel shows the residual distribution, helping assess the properties of the squared residuals.

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of σ^2. The calculated value in our case is $\widetilde{\sigma}^2 = 0.819646$. With this, and based on (68), we have that the estimated volatility is

$$\widehat{\sigma}_t^2 = \widetilde{\sigma}^2 e^{\widehat{h}_t}. \tag{75}$$

Figure 9 shows the estimated conditional variance, $\widetilde{\sigma}^2 \exp\{\widehat{h}_t\}$, for the entire period studied. It is important to note that there is a peak corresponding to October 22, 2008. At that time, between October 14 and 24 of that year (between observation 1403 and observation 1411) there was a sharp fall in the stock market, capital flight, and a significant rise in the value of the dollar. This is also seen when working with the ARCH-GARCH family of models, as can be seen in the upper part of Figure 4. Also observed in Figure 9 is a minor depression close to September 19, 2008 (observation 1387) corresponding to the rise in stocks on Wall Street due to the US bailout plan, a peak between June 16 and 26, 2014 (between observation 2781 and observation 2788) corresponding to an adverse ruling by the US Supreme Court on Argentina's debt, and others.

In order to detect possible relationships between the series studied and volatility, Figure 10 shows the estimated conditional standard deviation, i.e. $\widehat{\sigma}_t$, versus the standardized residuals of the model defined in the equation (66) for the Merval returns series for the period between January 13, 2003 and May 22, 2015. It is clear that no structure is observed that relates them.

In Figure 11 we can see, at the top, the last ten observations of the series under study, in red, and the corresponding predictions (in blue) of the conditional mean (model given in (66)), in the middle part are the residuals of that estimated model with the corresponding 95% confidence band that serve to determine if the residuals differ significantly from zero, so it is observed that they do not differ significantly from zero. In the lower part, these residuals are shown but standardized, that is, they are transformed so that they have a mean of zero and variance of one. With this we can say that the fit is adequate.

Figure 12 shows the prediction of the $\log\left(\widehat{\nu}_t^2\right)$ series where the $\widehat{\nu}_t$ are the residuals of the estimation of (66) in blue) versus the observed series of $\log\left(\widehat{\nu}_t^2\right)$ for the period between January 13, 2003 and May 22, 2015 (the ones corresponding to the last ten observations are shown) (top). Residuals of the adjustment of the model (70) (in blue) with a 95% confidence band (middle part) and the graph of these residuals after having been standardized. As we can see, it can be concluded that the adjustment is adequate.

7 FINAL CONSIDERATIONS

The series studied is made up of the first differences of the logarithm of the level of the Merval index. This is a stock market index calculated at the Buenos Aires Stock Exchange (BCBA), Argentina. It is a series with information corresponding to all working days of the stock market. The period analyzed goes from January 13, 2003 to May 22, 2015. There are 3006 observations. It covers a period in which there was no change in government affiliation. This eliminates the effects that could have been introduced into the market by changes in the governing group.

In our research, we set out to analyze methods for dealing with a wide variety of data with irregularities that occur in time series. Autoregressive integrated moving average models (or ARIMA models) are often considered to provide the main basis for modeling any time series. However, given the current state of development of time series research, there may be more attractive and, above all, more efficient alternatives. Many economic time series do not have a constant mean and also in most cases there are phases where relative calm reigns followed by periods of significant changes, i.e. variability changes over time. This behavior is what is called *volatility*.

To remedy this fact and to take into account the presence of volatility in an economic series, it is necessary to resort to models known as *conditional heteroscedastic models*. In these models, the variance of a series at a given point in time depends on past information and other data available up to that point in time, so a conditional variance must be defined, which is not constant and does not coincide with the overall variance of the observed series.

Among the models we have presented are the models of the ARCH family. The ARCH models or autoregressive models with conditional heteroscedasticity were first presented by Engle in 1982 with the objective of estimating the variance of inflation. The basic idea of this model is that y_t s not serially correlated but the volatility or conditional variance of the series depends on the past of the series by means of a quadratic function. However, these models are rarely used in practice due to their simplicity.

A good generalization of this model is found in the GARCH-type models introduced by Bollerslev (1986). This model is also a weighted average of a quadratic function of the past of the series, but it is more parsimonious than the ARCH-type models and even in its simplest form it has proven to be extremely successful in predicting conditional variances, so we decided to use them when working with our data.

The old saying "A painting worths more than thousand words" is quite true in the analysis of any set of information. Before applying any statistical method to the data under study, it is essential to observe it graphically in order to become familiar with it. This can have numerous benefits, as we have explained at the beginning of our analysis, since this process will serve as an indicator of ideas for a more detailed later study. This was the first step in our work in which we were able to see the main characteristics of the series and it helped us to make an appropriate adjustment to it.

Firstly, we decided to fit a GARCH-type model that captures the main characteristics of the data. We saw that it adequately takes into account the volatility of the series. With this analysis we have been able to capture some situations where volatility has a very important significance. We understand that the model used is adequate to predict the series and its components, in particular the volatility.

In the second part of this work it was decided to use a stochastic volatility approach to analyze the series under study, which turned out to be very useful in capturing the main characteristics of the data

The ARCH or GARCH family models assume that the conditional variance (volatility) depends on past values. In other words, and using the notation we saw above, if σ_t^2 is the volatility, the ARCH-GARCH family assumes that it depends on the

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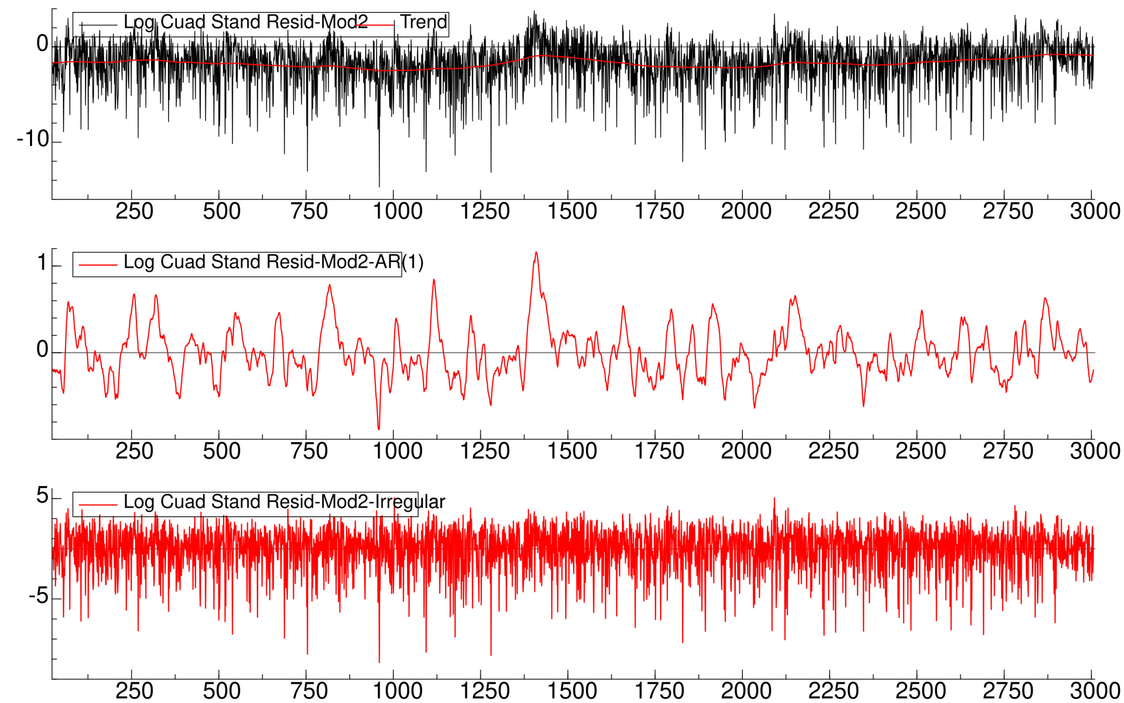
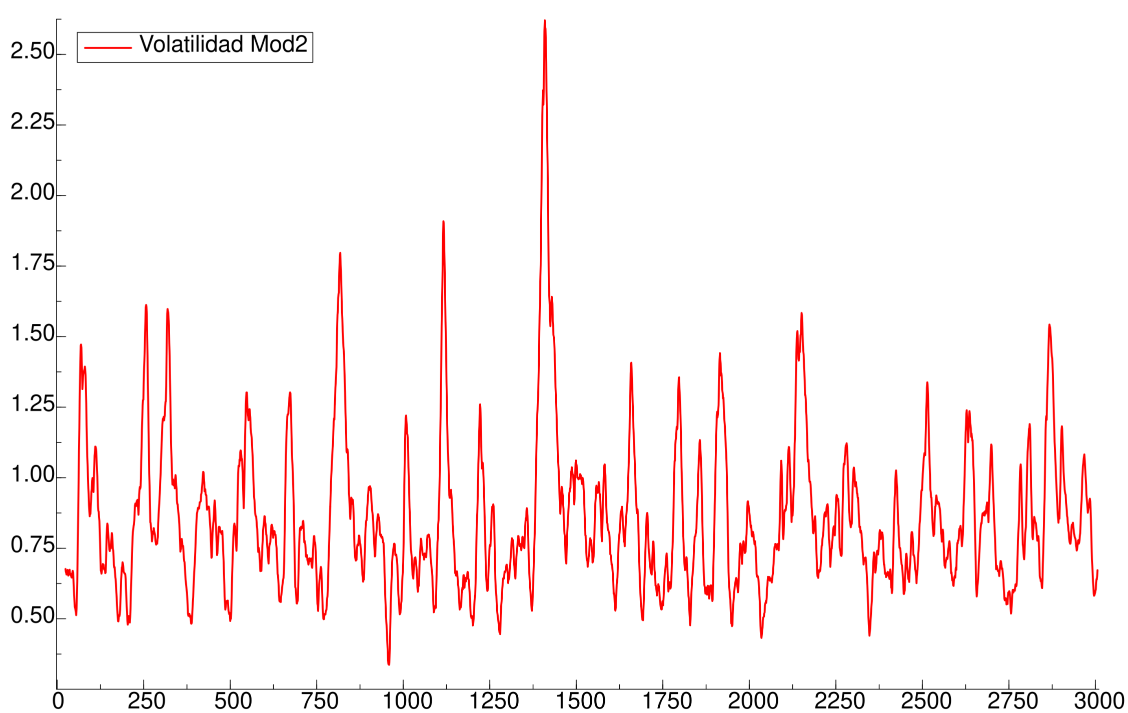
Figure 8. *Decomposition of the Log-Squared Residuals of the Merval Returns Model (2003-2015)*



Note. The figure presents the decomposition of the log-squared residuals of the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. The top panel shows the log-squared residuals with the level estimate. The middle panel displays the estimated AR(1) component, whose equation is given in Equation (62). The bottom panel presents the estimates of the irregular component, corresponding to Equation (61).

Figure 9. *Estimated Volatility of the Merval Returns (2003-2015)*



Note. The figure illustrates the estimated volatility of the Merval returns series from January 13, 2003, to May 22, 2015. The volatility is modeled using a conditional heteroskedasticity approach, capturing periods of high and low market fluctuations.

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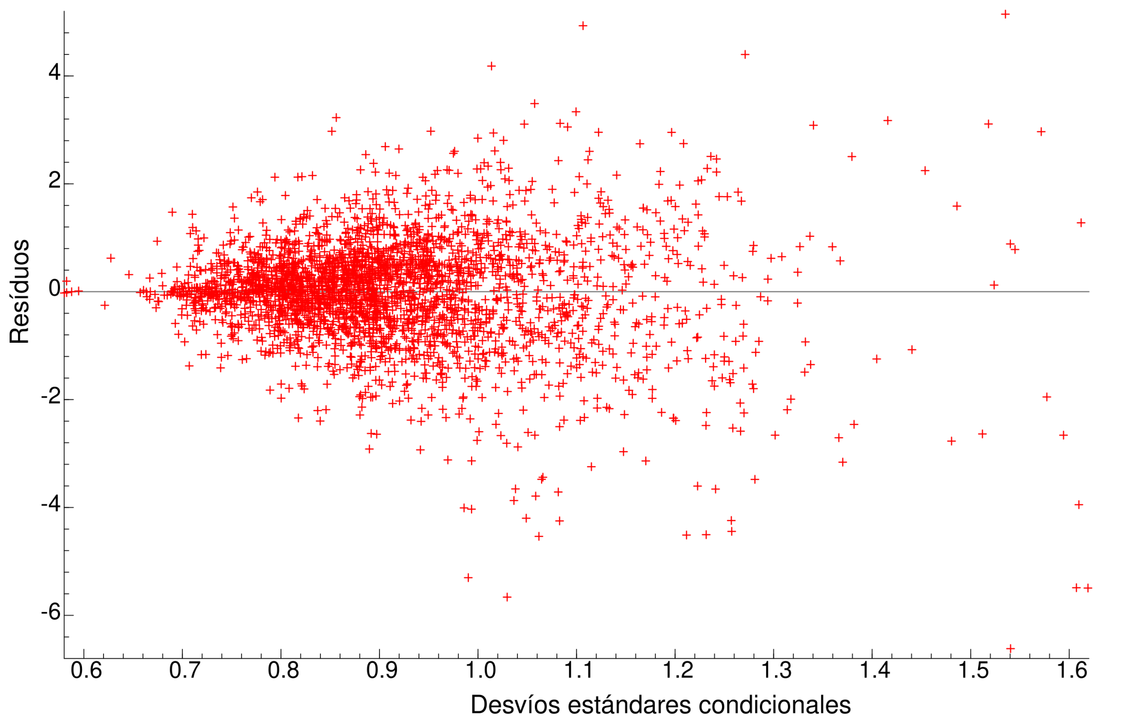
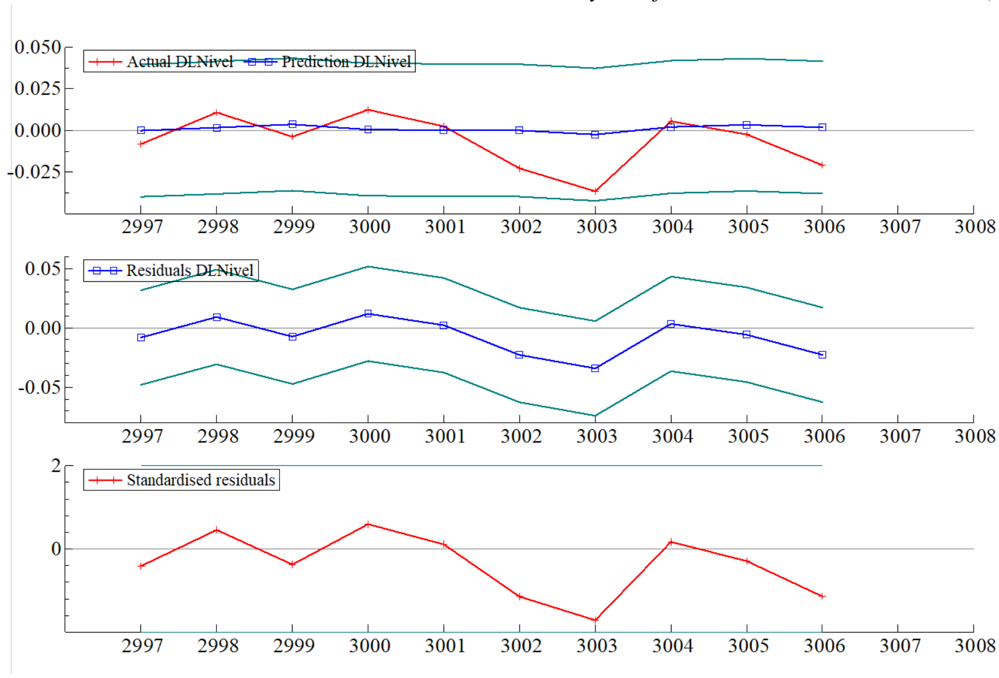
Figure 10. *Conditional Standard Deviation and Standardized Residuals of the Merval Returns Model (2003-2015)*



Note. The figure compares the estimated conditional standard deviation with the standardized residuals of the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. This comparison helps assess whether the standardized residuals exhibit homoskedasticity, validating the adequacy of the volatility model.

Figure 11. *Conditional Mean Prediction and Residual Analysis of the Merval Returns Model (2003-2015)*



Note. The figure presents the conditional mean prediction from the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. The top panel compares the predicted conditional mean (blue) with the observed returns (red), highlighting the last ten observations. The middle panel shows the model residuals with a 95% confidence band (blue). The bottom panel displays the standardized residuals for further evaluation of model adequacy.

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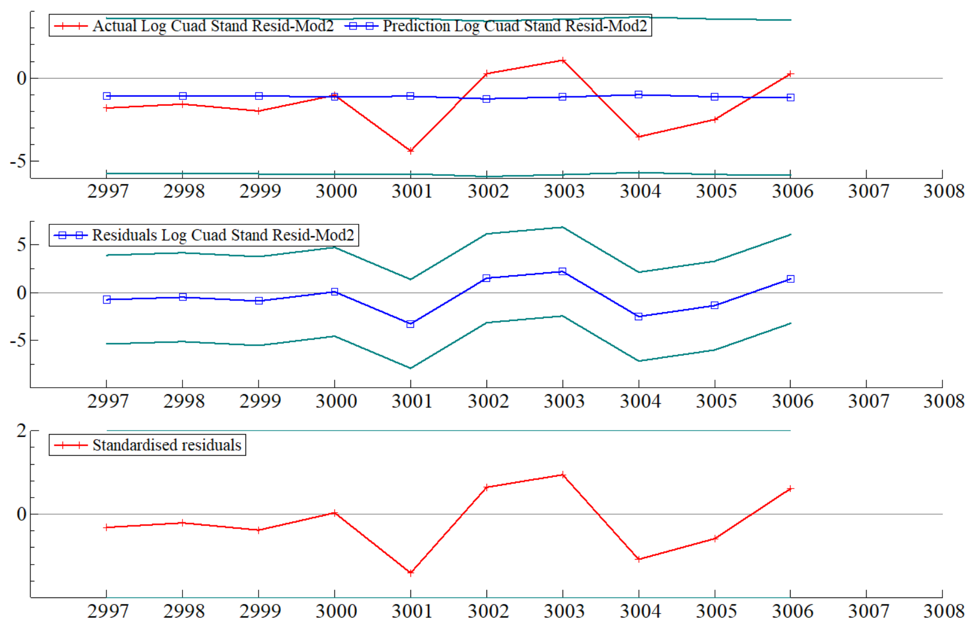
Figure 12. *Prediction of the Log-Squared Residuals of the Merval Returns Model (2003-2015)*



Note. The figure presents the prediction of the log-squared residuals ($\log(\widehat{\nu_t^2})$) from the model defined in Equation (66) for the Merval returns series from January 13, 2003, to May 22, 2015. The top panel compares the predicted values (blue) with the observed series (red), highlighting the last ten observations. The middle panel displays the residuals of the model fit in Equation (70) with a 95% confidence band (blue). The bottom panel presents the standardized residuals for further diagnostic analysis.

series y_j for $j < t$. On the other hand, the *stochastic volatility model* or SVM, proposed for the first time by Taylor (1980, 1986, 1994) does not start from this assumption. This model is based on the fact that the volatility σ_t^2 depends on its past values (σ_j^2 for $j < t$) but is independent of the past values of the series under analysis (y_j for $j < t$). Shephard and Pitt (1997) proposed the use of importance sampling to estimate the likelihood function in the non-Gaussian case. Since the MVE is a hierarchical model, Jaquier, Polson, and Rossi (1994) proposed a Bayesian analysis of it. See also Shephard (2005), Shephard and Pitt (1997), Kim, Shephard, and Chib (1998), and Ghysels, Harvey, and Renault (1996). An overview of the SVM estimation problem is given by Motta (2001).

As shown in Harvey, Ruiz, and Shephard (1994), the state-station form provides the basis for quasi-maximum likelihood estimation via the Kalman filter and smoother and also allows for constructing smoothed estimates of the variance component h_t and making predictions. One of the attractions of the quasi-maximum likelihood approach is that it can be applied without an assumption about a particular distribution for ε_t. Another attraction of using a quasi-maximum likelihood procedure via the Kalman filter and smoother to estimate SVM is that it can be carried out directly using standard computing packages such as STAMP by Koopman, Harvey, Doornik, and Shephard (2010). This is a great advantage compared to more labor-intensive simulation-based methods. Finally, by using an MVE, it was possible to estimate the different parts of the volatility (the scaling constant and the basic volatility).

Finally, we can say that both the ARCH-GARCH family models and the SVM work very well to estimate and predict volatility. But the SVMs have the advantage that both in the definition of their conditional mean given in the equations (54) and (55) and in the corresponding conditional variance given in the equations (56), (57) and (58) it is possible to introduce non-observable but estimable components such as trend, seasonality, cycles, structural changes, etc., and also explanatory variables, all due to the fact that these models are put in the form of a state space, which provides great generality when carrying out the work. Another important advantage of this approach is that it is estimated using a procedure based on quasi-maximum likelihood, which gives great flexibility to the procedure and a lot of robustness to the estimators.

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